

# Cardy-Verlinde formula in Taub-NUT/Bolt-(A)dS space

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We consider a finite action for a higher dimensional Taub-NUT/Bolt-(A)dS space via the so-called counter term subtraction method. In the limit of high temperature, we show that the Cardy-Verlinde formula holds for the Taub-Bolt-AdS metric and for the specific dimensional Taub-NUT-(A)dS metric, except for the Taub-Bolt-dS metric.

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## I. INTRODUCTION

The AdS/CFT duality was first conjectured by [1] in his search for relationship between gauge theories and strings. The AdS/CFT correspondence [2, 3, 4, 5, 6] asserts there is an equivalence between a gravitational theory in the bulk and a conformal field theory in the boundary. According to AdS/CFT, a  $(d+1)$ -dimensional S-(A)dS action  $A$  is given by

$$A = A_B + A_{\partial B} + A_{ct} \quad (1)$$

where the bulk action  $A_B$ , action boundary  $A_{\partial B}$ , and counterterm action  $A_{ct}$  are given as

$$\begin{aligned} A_B &= \frac{1}{16\pi G_{d+1}} \int_{\mathcal{M}} d^{d+1}x \sqrt{-g} (\mathcal{R} - 2\Lambda), \\ A_{\partial B} &= -\frac{1}{8\pi G_{d+1}} \int_{\partial\mathcal{M}} d^d x \sqrt{-\gamma} \Theta, \\ A_{ct} &= -\frac{1}{8\pi G_{d+1}} \int_{\partial\mathcal{M}} d^d x \sqrt{-\gamma} \left\{ -\frac{d-1}{l} \right. \\ &\quad - \frac{lR}{2(d-2)} \mathcal{F}(d-3) \\ &\quad - \frac{l^3}{2(d-2)^2(d-4)} \\ &\quad \times \left( R_{ab}R^{ab} - \frac{d}{4(d-1)} R^2 \right) \mathcal{F}(d-5) \\ &\quad + \frac{l^5}{(d-2)^3(d-4)(d-6)} \\ &\quad \times \left( \frac{3d+2}{4(d-1)} R R_{ab} R^{ab} - \frac{d(d+2)}{16(d-1)^2} R^3 \right. \\ &\quad + \frac{d-2}{2(d-1)} R^{ab} \nabla_a \nabla_b R - R^{ab} \square R_{ab} \\ &\quad \left. + \frac{1}{2(d-1)} R \square R \right) \mathcal{F}(d-7) + \dots \Big\}, \quad (2) \end{aligned}$$

where a negative cosmological constant  $\Lambda$  is  $\Lambda = -d(d-1)/2l^2$ ,  $\Theta$  is the trace of extrinsic curvature. Here,  $\mathcal{F}(d)$  is a step function, 1 when  $d \geq 0$ , 0 otherwise. The

boundary action  $A_{\partial B}$  is added to the action  $A$  to obtain equations of motion well behaved at the boundary. Then the boundary energy-momentum tensor is expressed in [7]

$$\frac{2}{\sqrt{-\gamma}} \frac{\delta A_{\partial B}}{\delta \gamma^{ab}} = \Theta_{ab} - \gamma_{ab} \Theta. \quad (3)$$

The counterterm action  $A_{ct}$  is added to the action  $A$  to remove the divergence appearing as the boundary goes to infinity [8]. For low dimensional S-AdS, a few terms in the counterterm action  $A_{ct}$  were explicitly evaluated in [8, 9]. Using the universality of the structure of divergences, the counterterm action  $A_{ct}$  for arbitrary dimension is suggested in [10]. This action  $A$  (1) leads to the entropy  $S$  via the Gibbs-Duhem relation

$$S = \frac{E}{T} - A \quad (4)$$

where  $T$  denotes the temperature and  $E$  is the total energy.

The entropy of the (1+1)-dimensional CFT is expressed in terms of the Virasoro operator  $L_0$  and the central charge  $c$ , the so-called the Cardy formula [11]. Using conformal invariance, the generalized Cardy formula in arbitrary dimension is shown to be given universal form as [12] (for the review articles of the issue, see, e.g., [13, 14, 15])

$$S_{\text{CFT}} = \frac{2\pi R}{\sqrt{ab}} \sqrt{E_c(2E - E_c)}, \quad (5)$$

where  $a$  and  $b$  are certain constants.  $R$  denotes the radius of the universe at a given time and  $E_c$  is the Casimir energy defined by

$$E_c = dE - (d-1)TS. \quad (6)$$

Employing AdS/CFT dual picture,  $\sqrt{ab}$  is fixed to  $(d-1)$  exactly, in particular, for a  $d$ -dimensional CFT on  $\mathbf{R} \times S^{d-1}$  [12]. Then, the entropy is given as

$$S_{\text{CFT}} = \frac{2\pi R}{d-1} \sqrt{E_c(2E - E_c)}, \quad (7)$$

which is shown to hold for Schwarzschild (A)dS (S-(A)dS) [12, 16], charged (A)dS [17, 18], Kerr-(A)dS [18, 19], and Taub-Bolt-AdS<sub>4</sub> [20]. There are many other relevant papers on the subject [21, 22, 23, 24, 25]. Thus,

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one may naively expect that the entropy of all CFTs that have an AdS-dual description is given as the form (7). However, AdS black holes do not always satisfy the Cardy-Verlinde formula (see, e.g., [17, 26]). Therefore, one intriguing question is whether this formula is valid for higher dimensional Taub-NUT-(A)dS at high temperature. In this Letter, we will endeavor to do this.

## II. TAUB-NUT/BOLT-ADS BLACK HOLE

When the total number of dimension of the spacetime is even,  $(d+1) = 2u+2$ , for some integer  $u$ , the Euclidean section of the arbitrary  $(d+1)$ -dimensional-Taub-NUT-AdS metric, for a  $U(1)$  fibration over a series of the space  $\mathcal{M}^2$  as the base space  $\bigotimes_{i=1}^u \mathcal{M}^2$ , is given by [27, 28, 29, 30, 31, 32, 33] (for the generalized versions of the issue, see, e.g., [34, 35])

$$ds_{\text{AdS}}^2 = f(r) \left[ dt_E + 2N \sum_{i=1}^u \cos(\theta_i) d\phi_i \right]^2 + \frac{dr^2}{f(r)} + (r^2 - N^2) \sum_{i=1}^u \left[ d\theta_i^2 + \sin^2(\theta_i) d\phi_i^2 \right], \quad (8)$$

where  $N$  represents a NUT charge for the Euclidean section, and the metric function  $f(r)$  has the general form

$$f(r) = \frac{r}{(r^2 - N^2)^u} \int^r \left[ \frac{(p^2 - N^2)^u}{p^2} + \frac{(2u+1)(p^2 - N^2)^{u+1}}{l^2 p^2} \right] dp - \frac{2mr}{(r^2 - N^2)^u}, \quad (9)$$

with a cosmological parameter  $l$  and a geometric mass  $m$ .

Requiring  $f(r)|_{r=N}$  the NUT solution occurs. Then for AdS spacetime the inverse of the temperature  $\beta$  arises from imposed condition in order to ensure regularity in the Euclidean time  $t_E$  and radial coordinate  $r$  [27, 28, 29, 30, 31, 32, 33]

$$\beta = \frac{4\pi}{f'(r)} \Big|_{r=N} = \frac{2(d+1)\pi N}{q}, \quad (10)$$

where  $\beta$  is the period of  $t_E$ . Here  $q$  is a positive integer, which originates from removing Misner string singularities. Using counter term subtraction method the regularized action is given as [27, 28, 29, 30, 31, 32, 33]

$$I_{\text{NUT}} = \frac{(4\pi)^{\frac{d}{2}} N^{d-2} \left( (d-1)N^2 - l^2 \right)}{32\pi^2 l^2} \times \Gamma\left(\frac{2-d}{2}\right) \Gamma\left(\frac{d+1}{2}\right) \beta. \quad (11)$$

Employing the thermal relation  $E = \partial_\beta I$  the total energy

can also be written by

$$E = \frac{(4\pi)^{\frac{d}{2}} (d-1) N^{d-2} \left( (d+1)N^2 - l^2 \right)}{32\pi^2 q l^2} \times \Gamma\left(\frac{2-d}{2}\right) \Gamma\left(\frac{d+1}{2}\right), \quad (12)$$

and the entropy is given as [27, 28, 29, 30, 31, 32, 33]

$$S_{\text{NUT,AdS}} = \frac{(4\pi)^{\frac{d}{2}} N^{d-2} \left( d(d-1)N^2 - (d-2)l^2 \right)}{32\pi^2 l^2} \times \Gamma\left(\frac{2-d}{2}\right) \Gamma\left(\frac{d+1}{2}\right) \beta, \quad (13)$$

by the Gibbs-Duhem relation  $S = \beta M - I$  where  $M$  denotes the conserved mass

$$M = \frac{(d-1)(4\pi)^{\frac{d}{2}}}{16\pi^{\frac{3}{2}}} m. \quad (14)$$

Substituting (10), (12), and (13) into (6), one gets the Casimir energy [12]

$$E_c = \frac{(4\pi)^{\frac{d}{2}} (d-1) N^{d-2} \left( dN^2 - l^2 \right)}{16\pi^2 q l^2} \times \Gamma\left(\frac{2-d}{2}\right) \Gamma\left(\frac{d+1}{2}\right). \quad (15)$$

From now on, for convenience we use  $l/z$  instead of the universe radius  $R$  in (7) since the AdS metric is always asymptotically taken to be [36]

$$ds^2 = \frac{l^2}{z^2} dz^2 + \frac{l^2}{z^2} g_{ab}(x, z) dx^a dx^b, \quad (16)$$

where the  $r = \infty$  is put to  $z = 0$ , and the roman indexes  $a$  and  $b$  refer to boundary coordinates. When  $1/\sqrt{ab}$  in the formula (7) is taken to be  $2/(d+1)(d-1)(d-2)$ , the CFT entropy is given as

$$S_{\text{CFT}} = \frac{4\pi l \sqrt{|E_c(2E - E_c)|}}{(d+1)(d-1)(d-2)}, \\ = \frac{(4\pi)^{\frac{d}{2}} |dN^2 - l^2| (-1)^{[\frac{d}{2}]} \Gamma\left(\frac{d+1}{2}\right) \Gamma\left(\frac{2-d}{2}\right)}{4\pi(d+1)(d-2)q}, \quad (17)$$

where  $[x]$  is the Gauss number (greatest integer less than or equal to  $x$ ). Here it seems that the difference from the standard Cardy-Verlinde formula (7) is due to the distinctive nature of NUT solution in AdS space like asymptotically locally AdS (ALAdS) metric. In the limit of high temperature,  $N \rightarrow 0$ , leading term in the entropy of CFT can be expressed as

$$S_{\text{CFT}} = \frac{(4\pi)^{\frac{d}{2}} (d+1)(d-2) N^{d-1}}{16\pi q} \times (-1)^{[\frac{d}{2}]} \Gamma\left(\frac{d+1}{2}\right) \Gamma\left(\frac{2-d}{2}\right) \\ = (-1)^{[\frac{d}{2}]} S_{\text{NUT,AdS}}. \quad (18)$$

This result shows that the entropy of the Taub-NUT-AdS space suffices to be the generalized Cardy-Verlinde formula (5) for all even  $u$  ( $d+1 = 2u+2$ ). This is reasonable because the Taub-NUT-AdS metric has the thermodynamically stable range depending on the magnitude of the NUT charge i.e. any NUT solution in AdS space for all odd  $u$  is thermodynamically unstable in the limit  $N \rightarrow 0$ .

Requiring  $f(r)|_{r=r_B > N}$  and  $f'(r)|_{r=r_B} = \frac{1}{(u+1)N}$ , the Bolt solution occurs. In Taub-Bolt-AdS metric, the inverse of the temperature, the total energy, and the entropy are respectively

$$\beta = \frac{4\pi}{f'(r)} \Big|_{r=r_B} = \frac{4\pi l^2 r_B}{l^2 + (2u+1)(r_B^2 - N^2)}, \quad (19)$$

$$E = \frac{(4\pi)^u u}{8\pi} \left( \sum_{k=0}^u \binom{u}{k} \frac{(-1)^k N^{2k} r_B^{2u-2k-1}}{2u-2k-1} + \sum_{k=0}^{u+1} \binom{u+1}{k} \frac{(-1)^k N^{2k} r_B^{2u-2k-1}}{2u-2k-1} \right), \quad (20)$$

$$S_{\text{Bolt,AdS}} = \frac{(4\pi)^u \beta}{16\pi l^2} \left[ \frac{(2u-1)(2u+1)(-1)^u N^{2u+2}}{r_B} + \sum_{k=0}^u \binom{u}{k} (-1)^k N^{2k} r_B^{2u-2k} \times \left( \frac{(2u-1)l^2}{(2u-2k-1)r_B} + \frac{(2u+1)(2u^2+3u-2k+1)r_B}{(2u-2k+1)(u-k+1)} \right) \right], \quad (21)$$

where  $r_B = \frac{ql^2 + \sqrt{q^2 l^4 + (2u+1)(2u+2)^2 N^2 [(2u+1)N^2 - l^2]}}{(2u+1)(2u+2)N}$ . The CFT entropy is written as

$$S_{\text{CFT}} = \frac{2\pi l \sqrt{E_c(2E - E_c)}}{2u\sqrt{2u-1}}, \quad (22)$$

where  $1/\sqrt{ab}$  is fixed to  $1/2u\sqrt{2u-1}$ . In the high temperature limit, the CFT entropy well suffices to be Cardy-Verlinde formula as the following

$$S_{\text{CFT}} = \frac{(4\pi)^u}{4N^{2u}} \left( \frac{ql^2}{2u^2+3u+1} \right)^{2u} = S_{\text{Bolt,AdS}}. \quad (23)$$

Note that the higher dimensional Taub-Bolt-AdS space follows the generalized Cardy-Verlinde formula (5) even if Taub-Bolt-AdS<sub>4</sub> space ( $u=1$ ) exactly satisfies the Cardy-Verlinde formula (7) [20].

### III. TAUB-NUT/BOLT-DS BLACK HOLE

Taub-NUT-dS metric is obtained from the Taub-NUT-AdS metric by replacing  $l^2 \rightarrow -l^2$ , and one has [37, 38,

39]

$$ds_{\text{dS}}^2 = -g(r) \left[ dt_E + 2N \sum_{i=1}^u \cos(\theta_i) d\phi_i \right]^2 - \frac{dr^2}{g(r)} + (r^2 - N^2) \sum_{i=1}^u \left[ d\theta_i^2 + \sin^2(\theta_i) d\phi_i^2 \right], \quad (24)$$

where  $g(r)$  is given as

$$g(r) = -\frac{r}{(r^2 - N^2)^u} \int^r \left[ \frac{(p^2 - N^2)^u}{p^2} - \frac{(2u+1)(p^2 - N^2)^{u+1}}{l^2 p^2} \right] dp + \frac{2mr}{(r^2 - N^2)^u} \quad (25)$$

Using parallel way as in the previous case, the inverse of the temperature, the total energy, the entropy, and the Casimir energy are obtained

$$\beta = \frac{4\pi}{g'(r)} \Big|_{r=N} = \frac{2(d+1)\pi|N|}{q}, \quad (26)$$

$$E = \frac{(4\pi)^{\frac{d}{2}}(d-1)N^{d-2} \left( (d+1)N^2 + l^2 \right)}{32\pi^2 q l^2} \times \Gamma\left(\frac{2-d}{2}\right) \Gamma\left(\frac{d+1}{2}\right), \quad (27)$$

$$S_{\text{NUT,AdS}} = \frac{(4\pi)^{\frac{d}{2}} N^{d-2} \left( d(d-1)N^2 + (d-2)l^2 \right)}{32\pi^2 l^2} \times \Gamma\left(\frac{2-d}{2}\right) \Gamma\left(\frac{d-1}{2}\right) \beta, \quad (28)$$

$$E_c = \frac{(4\pi)^{\frac{d}{2}}(d-1)N^{d-2} \left( dN^2 + l^2 \right)}{16\pi^2 q l^2} \times \Gamma\left(\frac{2-d}{2}\right) \Gamma\left(\frac{d+1}{2}\right). \quad (29)$$

In this case,  $1/\sqrt{ab}$  in (7) is fixed to  $2/(d+1)(d-1)(d-2)$ . Then, the entropy in boundary CFT is expressed as

$$S_{\text{CFT}} = \frac{4\pi l \sqrt{E_c(2E - E_c)}}{(d+1)(d-1)(d-2)}, = \frac{(4\pi)^{\frac{d}{2}}(dN^2 + l^2)(-1)^{[\frac{d}{2}]} \Gamma\left(\frac{d+1}{2}\right) \Gamma\left(\frac{2-d}{2}\right)}{4\pi(d+1)(d-2)q}. \quad (30)$$

In high temperature limit, leading term in the entropy of the CFT for all even  $u$  is precisely matched with that in the entropy of the Taub-NUT-dS space as the following

$$S_{\text{CFT}} = \frac{(4\pi)^{\frac{d}{2}}(d+1)(d-2)N^{d-1} \Gamma\left(\frac{4-d}{2}\right) \Gamma\left(\frac{3+d}{2}\right)}{16\pi^{\frac{3}{2}} q} = S_{\text{NUT,dS}}. \quad (31)$$

For the Bolt solution in dS space, the inverse of the temperature, the total energy, and the entropy are respectively

$$\beta = \frac{4\pi}{f'(r)} \Big|_{r=r_B} = -\frac{4\pi l^2 r_B}{l^2 + (2u+1)(r_B^2 - N^2)}, \quad (32)$$

$$E = -\frac{(4\pi)^u u}{8\pi} \left( \sum_{k=0}^u \binom{u}{k} \frac{(-1)^k N^{2k} r_B^{2u-2k-1}}{2u-2k-1} + \sum_{k=0}^{u+1} \binom{u+1}{k} \frac{(-1)^k N^{2k} r_B^{2u-2k-1}}{2u-2k-1} \right), \quad (33)$$

$$S_{\text{Bolt,dS}} = \frac{(4\pi)^u \beta}{16\pi l^2} \left[ -\frac{(2u-1)(2u+1)(-1)^u N^{2u+2}}{r_B} + \sum_{k=0}^u \binom{u}{k} (-1)^k N^{2k} r_B^{2u-2k} \times \left( -\frac{(2u-1)l^2}{(2u-2k-1)r_B} + \frac{(2u+1)(2u^2+3u-2k+1)r_B}{(2u-2k+1)(u-k+1)} \right) \right], \quad (34)$$

where  $r_B = \frac{ql^4 + \sqrt{q^2 l^2 + (2u+1)(2u+2)^2 N^2 [(2u+1)N^2 + l^2]}}{(2u+1)(2u+2)N}$ . The CFT entropy is given as

$$\frac{2\pi l \sqrt{|E_c|(2E - E_c)}}{2u\sqrt{2u-1}}, \quad (35)$$

where  $1/\sqrt{ab}$  is fixed to  $1/2u\sqrt{2u-1}$ . As the NUT charge goes to 0, the CFT entropy becomes

$$S_{\text{CFT}} = \frac{(4\pi)^u}{4N^{2u}} \left( \frac{ql^2}{2u^2 + 3u + 1} \right)^{2u} = -S_{\text{Bolt,dS}}, \quad (36)$$

which shows that no entropy of the Taub-Bolt-dS metric satisfies the Cardy-Verlinde formula. This means that any Bolt solution in dS space is thermodynamically unstable at high temperature limit.

#### IV. CONCLUSION

We have considered that the Taub-NUT/Bolt-(A)dS metric in general even dimension, and have checked that its metric suffices to be the the Cardy-Verlinde formula. In the limit of high temperature, we showed that the Taub-Bolt-AdS space well follows the generalized Cardy-Verlinde formula (7) rather than the Cardy-Verlinde formula (5). It seems that the modification of the standard Cardy-Verlinde formula (5) is due to the distinctive property of the Taub-NUT solution such as the ALAdS metric. It was proven that the leading term of the CFT entropy at the boundary for all even  $u$  is exactly matched with that of the entropy in the Taub-NUT-(A)dS space by using the generalized Cardy-Verlinde formula at high temperature. Thermal stability of the Taub-NUT-(A)dS solution for all odd  $u$  and Taub-Bolt-dS solution for all  $u$  is determined by the magnitude of the NUT charge so that the negative entropy occurs as the NUT charge goes to 0. Finally, the the breaking of Cardy-Verlinde formula in the Taub-Bolt-dS metric seems to reflect the fact that there is no Bolt solution in dS space due to the absence of hyperbolic NUT in AdS space [39].

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